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## Comment on “Constraining the smoothness parameter and dark energy using observational $H(z)$ data”

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**Abstract** In this Comment we discuss a recent analysis by Yu et al. [RAA 11, 125 (2011)] about constraints on the smoothness  $\alpha$  parameter and dark energy models using observational  $H(z)$  data. It is argued here that their procedure is conceptually inconsistent with the basic assumptions underlying the adopted Dyer-Roeder approach. In order to properly quantify the influence of the  $H(z)$  data on the smoothness  $\alpha$  parameter, a  $\chi^2$ -test involving a sample of SNe Ia and  $H(z)$  data in the context of a flat  $\Lambda$ CDM model is reanalyzed. This result is confronted with an earlier approach discussed by Santos et al. (2008) without  $H(z)$  data. In the  $(\Omega_m, \alpha)$  plane, it is found that such parameters are now restricted on the intervals  $0.66 \leq \alpha \leq 1.0$  and  $0.27 \leq \Omega_m \leq 0.37$  within 95.4% confidence level ( $2\sigma$ ), and, therefore, fully compatible with the homogeneous case. The basic conclusion is that a joint analysis involving  $H(z)$  data can indirectly improve our knowledge about the influence of the inhomogeneities. However, this happens only because the  $H(z)$  data provide tighter constraints on the matter density parameter  $\Omega_m$ .

**Key words:** cosmology — dark energy — cosmological parameters

It is widely known that the Universe is homogeneous and isotropic only at very large scales ( $\gtrsim 100 Mpc$ ). In moderate and smaller scales, the Universe is inhomogeneous. Since the light propagation probes the local gravitational field, the clumpiness of matter may affect the determination of physical parameters comparatively to the standard Friedmann-Robertson-Walker (FRW) geometry. Zeldovich (1964) and Kantowski (1969) were the first to study this kind of effect. Later on, Dyer & Roeder (1972, 1973) introduced the smoothness parameter  $\alpha$  to quantify the effect of the inhomogeneities in the magnification of a light beam. For  $\alpha = 0$  (empty beam), all the matter is clumped and for  $\alpha = 1$  the homogeneous case is recovered. Therefore, the smoothness parameter is restricted over the interval  $[0, 1]$ . For a clumpy Universe ( $\alpha \neq 1$ ), a new distance is derived which is sometimes called the Dyer-Roeder distance (Schneider et al., 1992).

Efforts to obtain observational bounds over  $\alpha$  were initially based on supernovae type Ia (SNe Ia) (Santos et al., 2008) and compact radio sources (Alcaniz, Lima & Silva 2004; Santos & Lima 2008). In particular, by assuming that the dark energy is a smooth component, Santos et al. (2008) obtained  $\alpha \geq 0.42$  within 95.4% confidence level with basis on the Riess et al. (2007) SNe Ia sample. It was also shown that compact radio sources (Gurvits et al., 1999; Lima & Alcaniz, 2000, 2002) did not constrain  $\alpha$  (Alcaniz et al., 2004; Santos & Lima, 2008).

Recently, Yu et al. (2011) claimed that better constraints over the Dyer-Roeder parameter  $\alpha$  could be obtained based only on the observational  $H(z)$  data. By using a  $\chi^2$  minimization method they found  $\alpha = 0.81^{+0.19}_{-0.20}$  and  $\Omega_M = 0.32^{+0.12}_{-0.06}$  at  $1\sigma$  confidence level. Further, by assuming a Gaussian prior of  $\Omega_M = 0.26 \pm 0.1$ , the limits  $\alpha = 0.93^{+0.07}_{-0.19}$  and  $\Omega_M = 0.31^{+0.06}_{-0.05}$  were also derived. Finally, for a  $\Lambda$ CDM model, the smoothness parameter was constrained to  $\alpha \geq 0.80$  with  $\omega$  weakly constrained around -1, where  $\omega$  describes the equation of the state of the dark energy ( $p_X = \omega\rho_X$ ). However, as it will be argued in the present comment, there is a profound contradiction between their implementation of the observational Hubble data and the underlying assumptions of the Dyer-Roeder approach. In other words, the  $H(z)$  data alone cannot constrain the  $\alpha$  parameter.

To begin with, let us first discuss the basic assumptions of the Dyer-Roeder procedure. The main hypothesis is that the Universe is locally inhomogeneous, where underdensities in voids are compensated by overdensities in clumps thereby making the Universe homogeneous at very large scales. A typical line of sight is far from the clumps, not suffering from gravitational lensing effects, being reasonable to consider that the light beam experiences an effective  $\alpha\rho_m$  matter energy density and negligible shear. On the other hand, the dynamics is expected to feel the influence of a volume smoothed description (Bildhauer & Futamase, 1991; Buchert & Ehlers, 1997; Linder, 1998) and it is the same as the homogeneous case. Thus, in the Dyer-Roeder approach, the Hubble parameter does not depend on the smoothness parameter. Actually, the homogeneous Hubble parameter is used in the derivation of the Dyer-Roeder equation, as can be seen in the following differential equation for the angular diameter distance (see, for instance, Mattsson, 2010)

$$H(z)\frac{d}{dz}\left[(1+z)^2H(z)\frac{d}{dz}d_A(z)\right] + 4\pi G[\rho(z) + p(z)]d_A(z) = 0. \quad (1)$$

In the above expression,  $H(z)$  stands for the Hubble parameter,  $d_A(z)$  for the angular diameter distance,  $\rho(z)$  for the total energy density and  $p(z)$  for the total pressure. The smoothness parameter enters only in the second term through the effective  $\rho(z)$  function.

On the other hand, in order to implement the observational Hubble data, Yu et al. (2011) also deduced the correspondence between  $H(z)$  and  $d_A(z)$  (see their Eqs. (22)-(25))

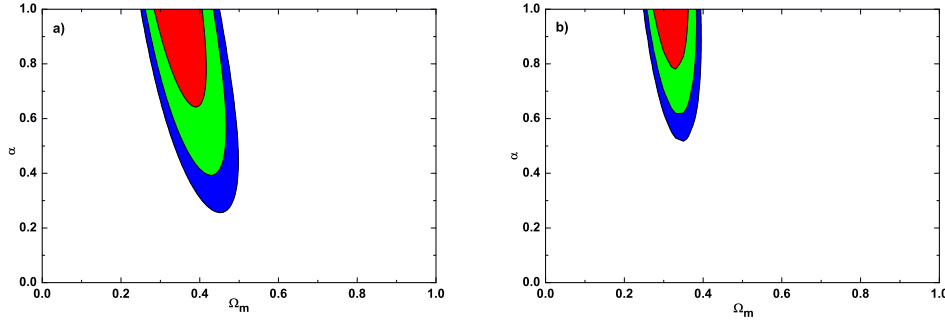
$$\frac{H(z)}{H_0} = \frac{1}{(1+z)d'_A(z) + d_A(z)}, \quad (2)$$

where  $H_0$  is the Hubble constant and the prime denotes differentiation with respect to  $z$ . However, as one may check, the above relation is valid only when  $\alpha = 1$ , that is, in the homogeneous case. In this way, it does not make sense to use a form for  $H(z)$  independent of  $\alpha$  to obtain the Dyer-Roeder distance (see Eq. 1), and, simultaneously, to consider  $H(z)$  with a dependence on  $\alpha$  as given by the above equation 2. It thus follows that the analysis made by Yu et al. (2011) is both conceptually and mathematically flawed and, as such, their corresponding results are meaningless.

Nevertheless, the question posed by Yu et al. (2011) concerning a possible influence of the  $H(z)$  data on the constraints of  $\alpha$  can still be considered at least in the context of a joint analysis, for instance, involving supernovas and other cosmological tests. In this case, since the observational Hubble data constrain the cosmology itself, it is interesting to quantify how the  $H(z)$  data can modify the limits established on the smoothness parameter using, for instance, the SNe type Ia analysis appearing in the paper by Santos et al. (2008). Naturally, one may think that the effect must be small because the  $H(z)$  does not depend explicitly on the value of  $\alpha$ .

In figure 1a we display the results obtained by Santos et al. (2008) through a  $\chi^2$ -test by using only the 182 SNe Ia from Riess et al. (2007) (gold sample). In their analysis they obtained  $0.42 \leq \alpha \leq 1.0$  and  $0.25 \leq \Omega_m \leq 0.44$  at  $2\sigma$  confidence level. The corresponding best fits are  $\alpha = 1$  and  $\Omega_m = 0.33$  and, therefore, fully in agreement with the homogeneous case.

In figure 1b, we show the present joint analysis by considering the same gold sample plus 12  $H(z)$  data (Simon et al., 2005; Daly et al., 2008). The parameters are now restricted over the intervals  $0.66 \leq \alpha \leq 1.0$  and  $0.27 \leq \Omega_m \leq 0.37$  within  $2\sigma$  confidence level. The best fits are  $\alpha = 1.0$  and  $\Omega_m = 0.32$ .



**Fig. 1** **a)** The  $\alpha - \Omega_m$  plane for 182 SNe Ia from Riess et al. (2007) as discussed by Santos et al. (2008). **b)** Constraints for a joint analysis involving the same SNe Ia sample plus 12  $H(z)$  data from Simon et al. (2005) and Daly et al. (2008). The constraints obtained with the joint analysis are  $0.66 \leq \alpha \leq 1.0$  and  $0.27 \leq \Omega_m \leq 0.37$  ( $2\sigma$ ), with a best fit of  $\alpha = 1.0$  and  $\Omega_m = 0.32$  (see comments in the text).

As should be physically expected, the constraints are mildly improved by introducing the  $H(z)$  data. In particular, the best fit value of the smoothness parameter is given by the homogeneous case ( $\alpha = 1$ ), as derived earlier by Santos et al. (2008). This fact can be understood by realizing that only the value of  $\Omega_m$  is directly constrained by the  $H(z)$  data. Naturally, the present results also suggest that the remaining analysis for  $\Lambda$ CDM models studied by Yu et al. (2011) should also be rediscussed.

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